Studies of Emittance Growth and Halo Particle Production in Intense Charged Particle Beams Using the Paul Trap Simulator Experiment

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Outline

- The analogy between the Paul Trap Simulator Experiment (PTSX) and Alternating-Gradient transport systems
- Random noise-induced emittance growth and halo particle production
- Adiabatic transverse beam compression with minimal emittance growth
- Focusing-Off-Defocusing-Off (FODO) waveforms for lower initial emittance

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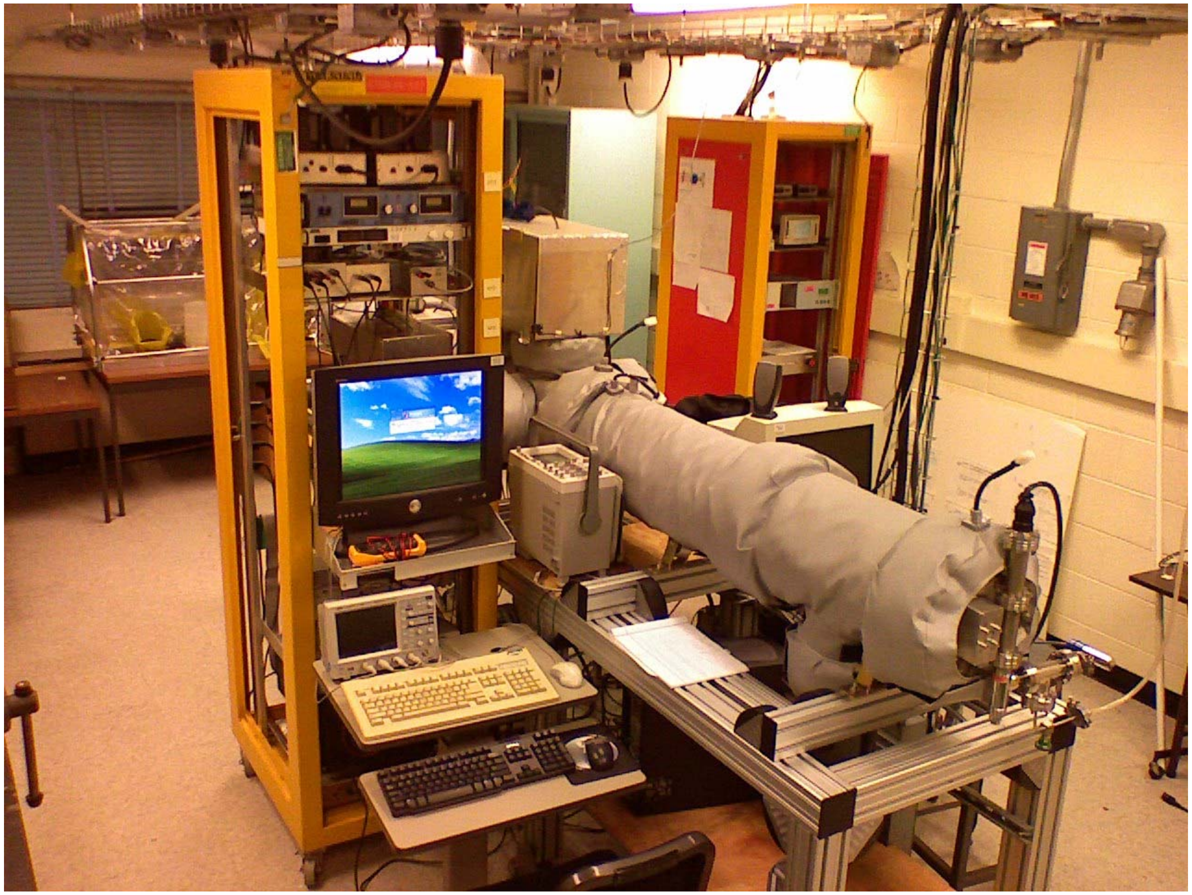
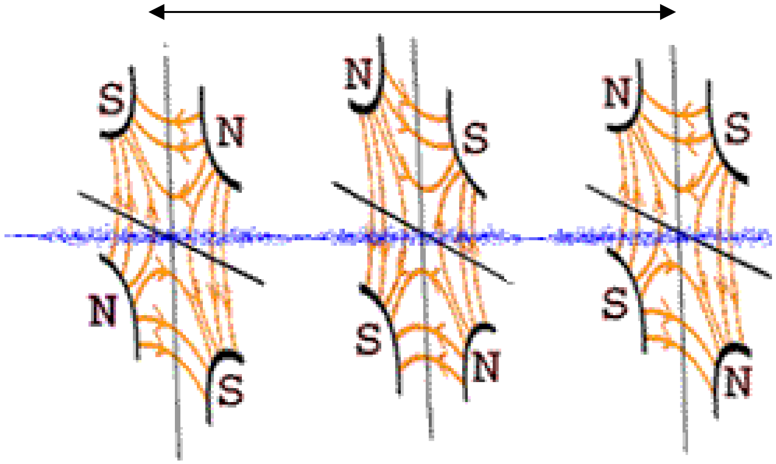
PTSX Simulates Nonlinear Beam Dynamics in Magnetic Alternating-Gradient Systems

- Purpose: PTSX simulates, in a compact experiment, the transverse nonlinear dynamics of intense beam propagation over large distances through magnetic alternating-gradient transport systems.
- Applications: Accelerator systems for high energy and nuclear physics applications, heavy ion fusion, spallation neutron sources, and high energy density physics.



Alternating-Gradient Transport Systems Use a Spatially Periodic Lattice of Quadrupole Magnets for Transverse Confinement

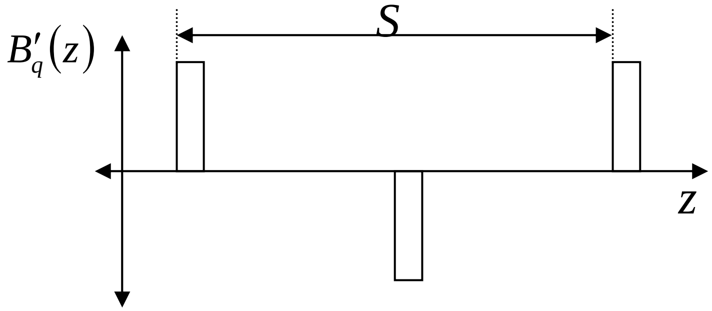


Focusing-Off-Defocusing-Off (FODO) Lattice

$$\boxed{\begin{aligned} \boldsymbol{B}_q^{foc}(\boldsymbol{x}) &= B_q'(z)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right) \\ \boldsymbol{F}_{foc}(\boldsymbol{x}) &= -\kappa_q(z)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right) \end{aligned}}$$

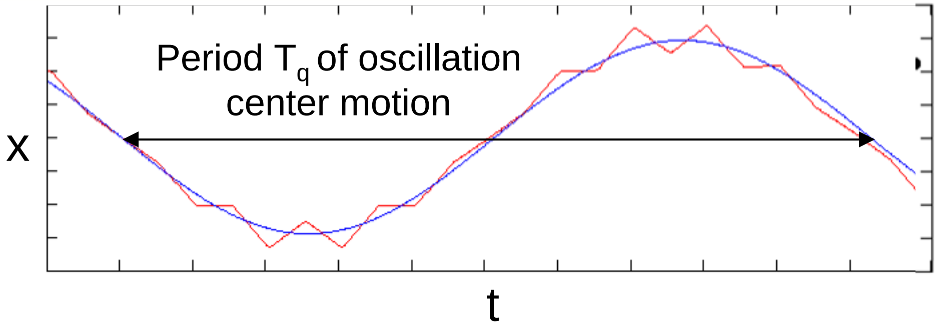
$$\boxed{\kappa_q(z) \equiv \frac{ZeB_q'(z)}{\gamma m\beta c^2}}$$

Example of FODO lattice



Average Transverse Focusing Frequency and Phase Advance Characterize the Motion – Emittance is a Measure of Beam Quality

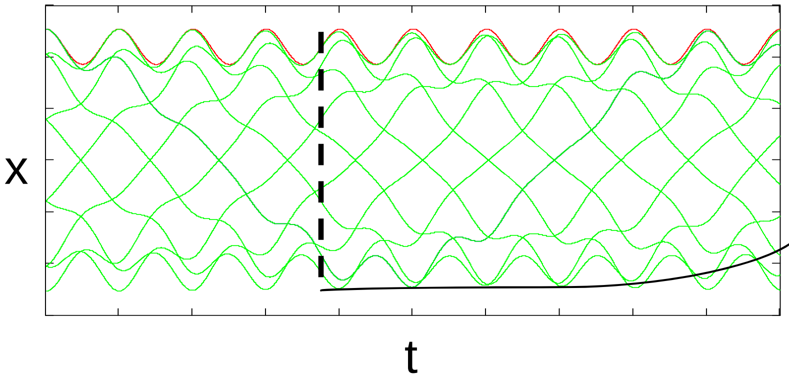
$\kappa_q(z)$: FODO lattice



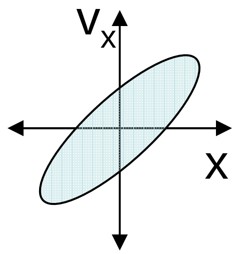
$\omega_q = 2\pi/T_q$ is the average transverse focusing frequency

$\sigma_v = \omega_q/f$ Here, the vacuum phase advance, σ_v, is 35°.

$\kappa_q(z)$: sinusoidal lattice

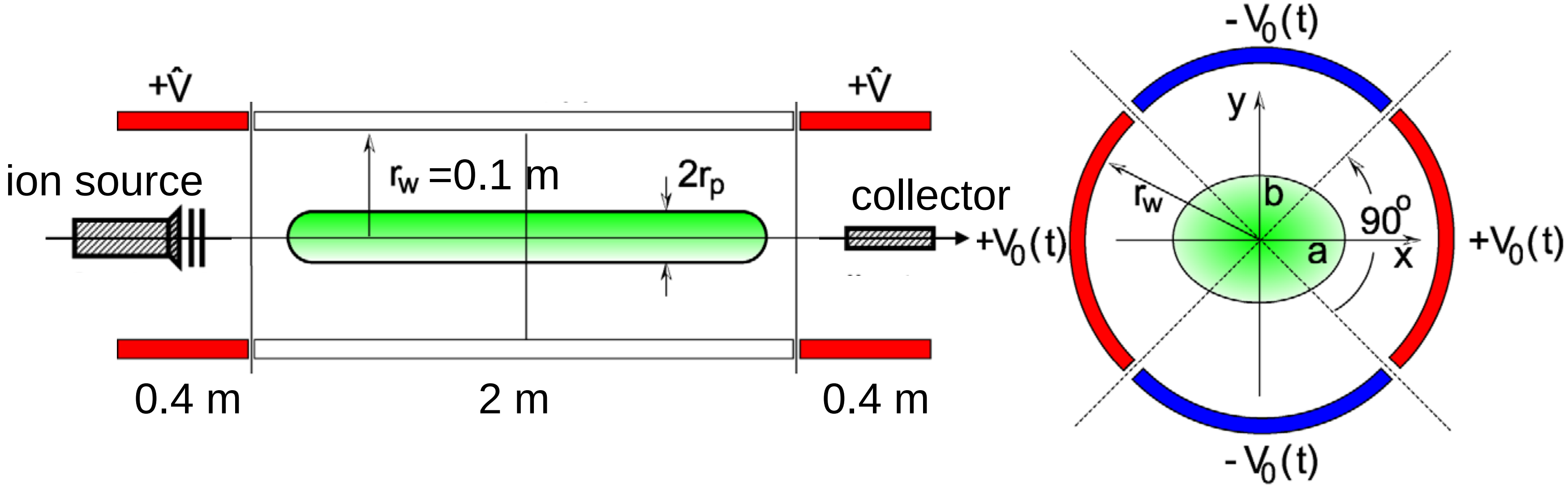


The (x,v_x) values of particles at the location of the dotted line gives an ellipse in phase space.



Emittance is the phase space area of the beam and scales as $R_b\ (kT)^{1/2}$

PTSX is a Linear Paul Trap that Confines a Pure Cesium Ion Plasma



$$e_b\phi_{ap}(x,y,t) = \frac{1}{2}\kappa_q(t)(x^2 - y^2)$$

$$\kappa_q(t) = \frac{8e_bV_0(t)}{m_b\pi\ r_w^2}$$

$$V_0(t) = V_{0\ \max}\sin(\omega t)$$

$$\omega_q = \frac{8e_bV_{0\ \max}}{m_b\pi\ r_w^2\ f}\frac{1}{2\sqrt{2}\pi}$$

Typical Parameters

V_0 = 150 V

f = 60 kHz

σ_v = 50°

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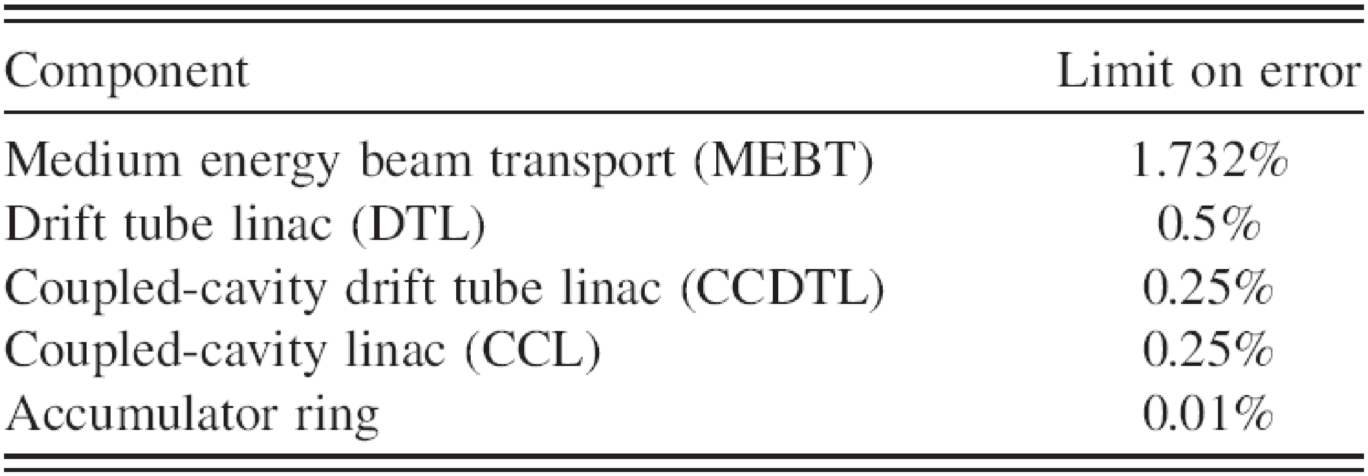
Field Errors Can Degrade Beam Quality – Strict Limits are Placed on Allowable Errors

Example: Spallation Neutron Source (SNS)

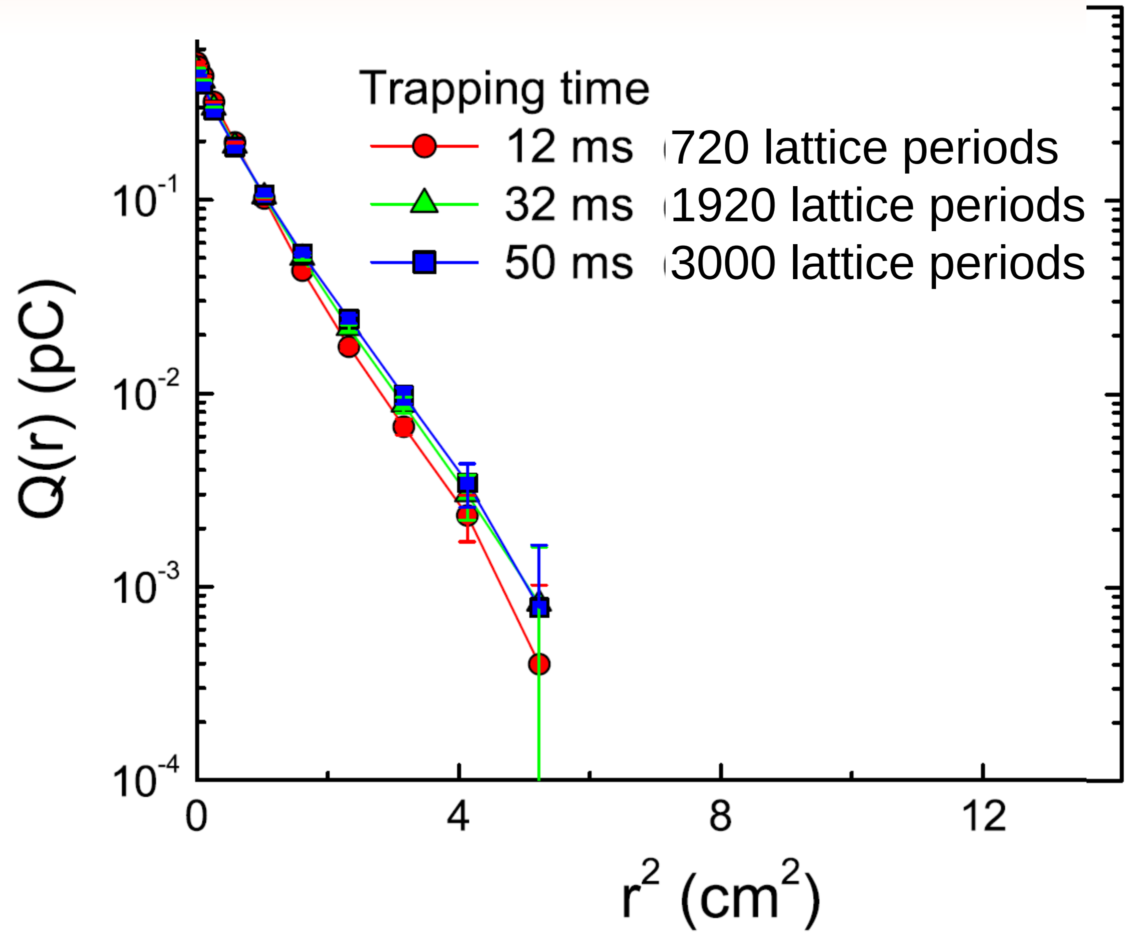
Values of quadrupole gradient error limits for various components of the SNS linac and accumulator ring.



Component	Limit on error
Medium energy beam transport (MEBT)	1.732%
Drift tube linac (DTL)	0.5%
Coupled-cavity drift tube linac (CCDTL)	0.25%
Coupled-cavity linac (CCL)	0.25%
Accumulator ring	0.01%

S. Henderson *et al.*, SNS Parameters List (SNS-100000000-PL0001-R13) ORNL (2005).

Without Applied Noise the Plasma is Stable for Over 3000 Lattice Periods



From Q(r), the line charge *N* and the root-mean-squared radius R_b are computed.

$$m\omega_q^2 R_b^2 = 2\bar{T}_\perp + \frac{Nq^2}{4\pi\epsilon_0}$$

$$\epsilon(t) = 2R_b\left(\omega_q^2 R_b^2 - \frac{Nq^2}{4\pi\epsilon_0 m}\right)^{1/2}$$

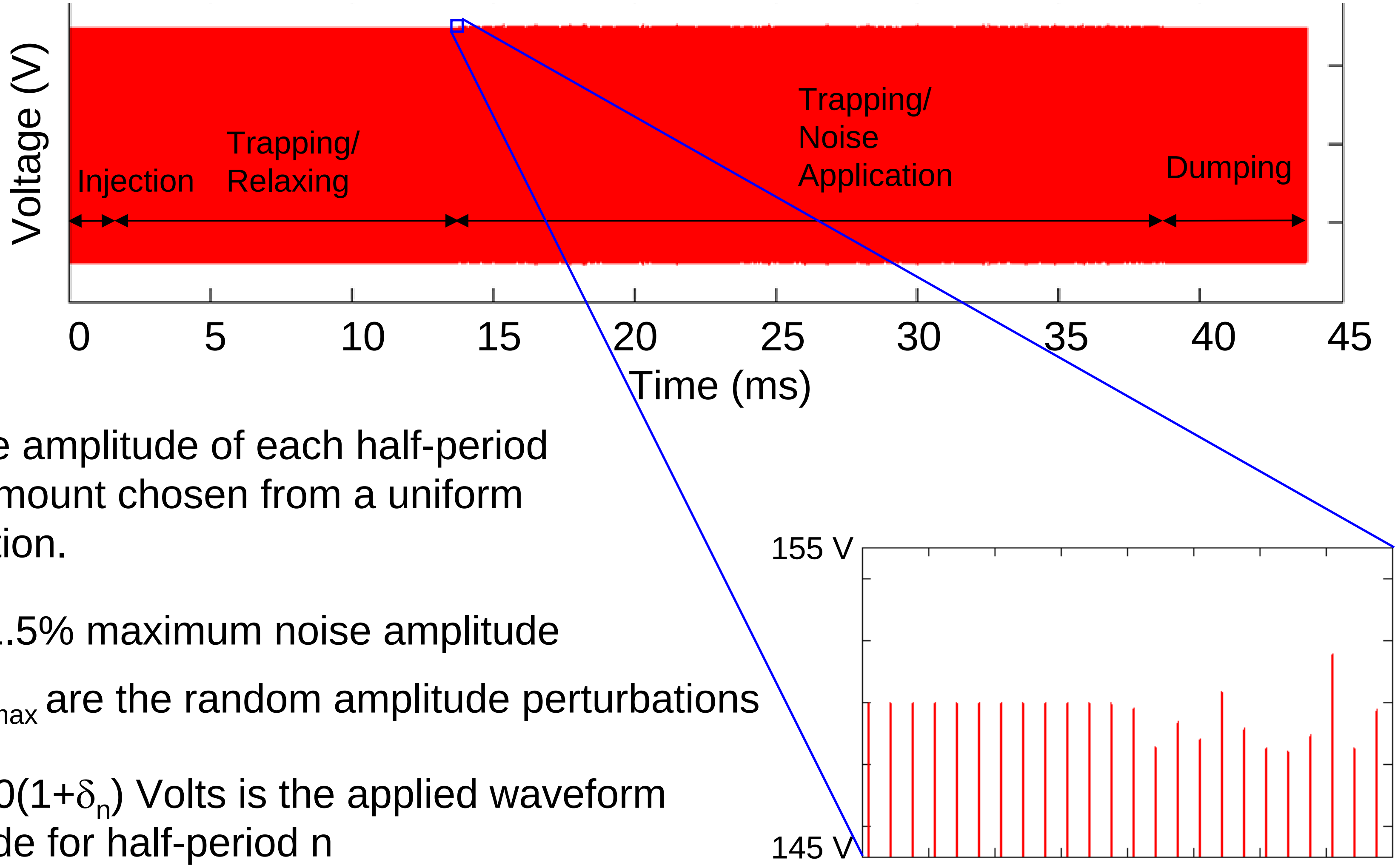
Normalized intensity parameter *s* is computed from Q(0)

$s = \omega_p^2/2\omega_q^2$

$0 < s < 1$

$s = 0.2$ here

Typical PTSX Applied Waveform to Explore Noise Effects



Vary the amplitude of each half-period by an amount chosen from a uniform distribution.

Δ_{max} = 1.5% maximum noise amplitude

$|\delta_n| < \Delta_{max}$ are the random amplitude perturbations

$V_n = 150(1+\delta_n)$ Volts is the applied waveform amplitude for half-period n

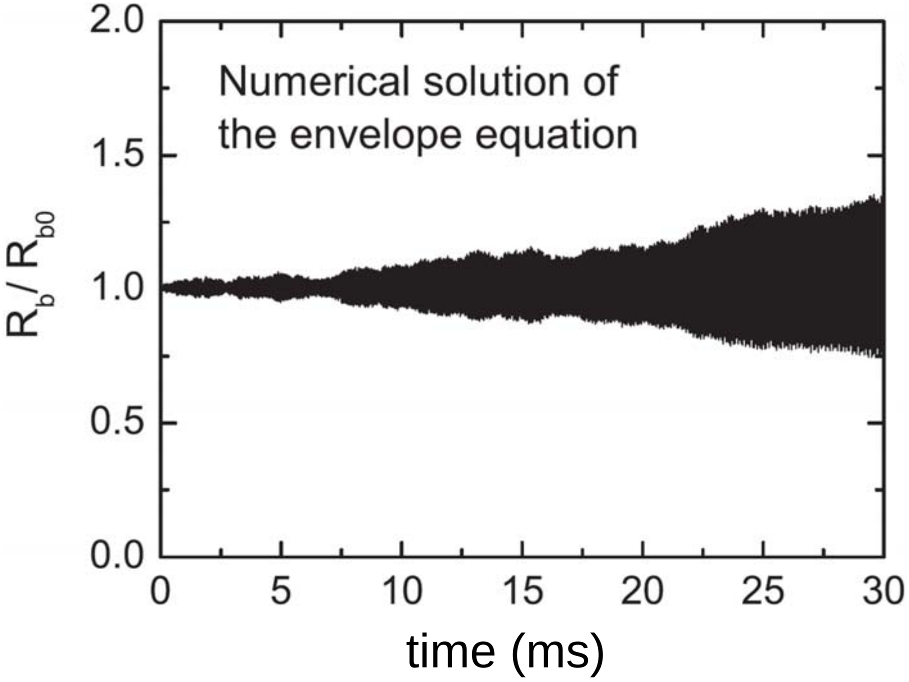
M. Chung *et al.*, Phys. Rev. ST Accel. Beams **12**, 054203 (2009).

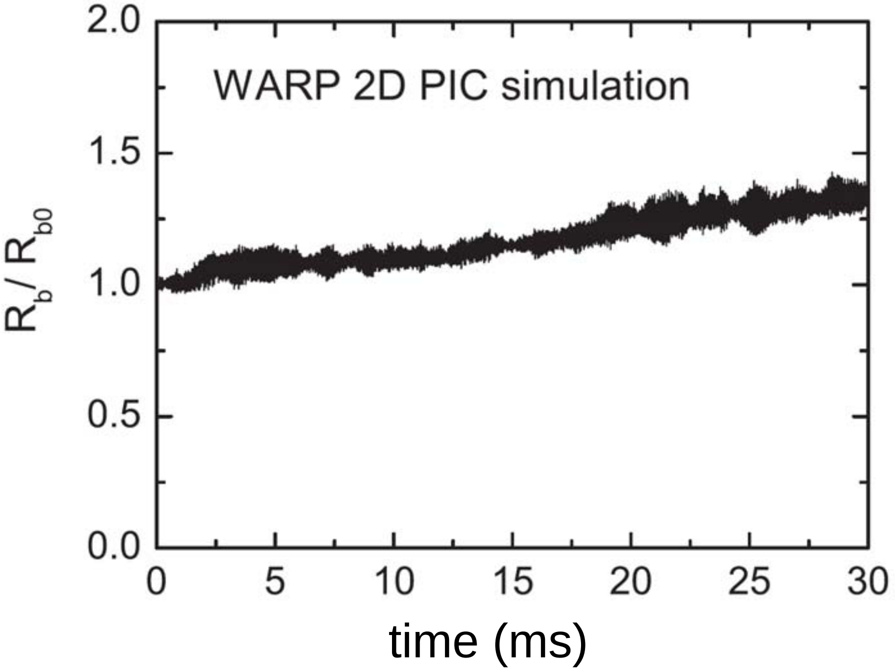
The Constant-Emittance Envelope Equation Predicts Growing Oscillations While WARP 2D PIC Simulations Predict Broadening of the Plasma

1% Noise Amplitude

Same error sample in both cases

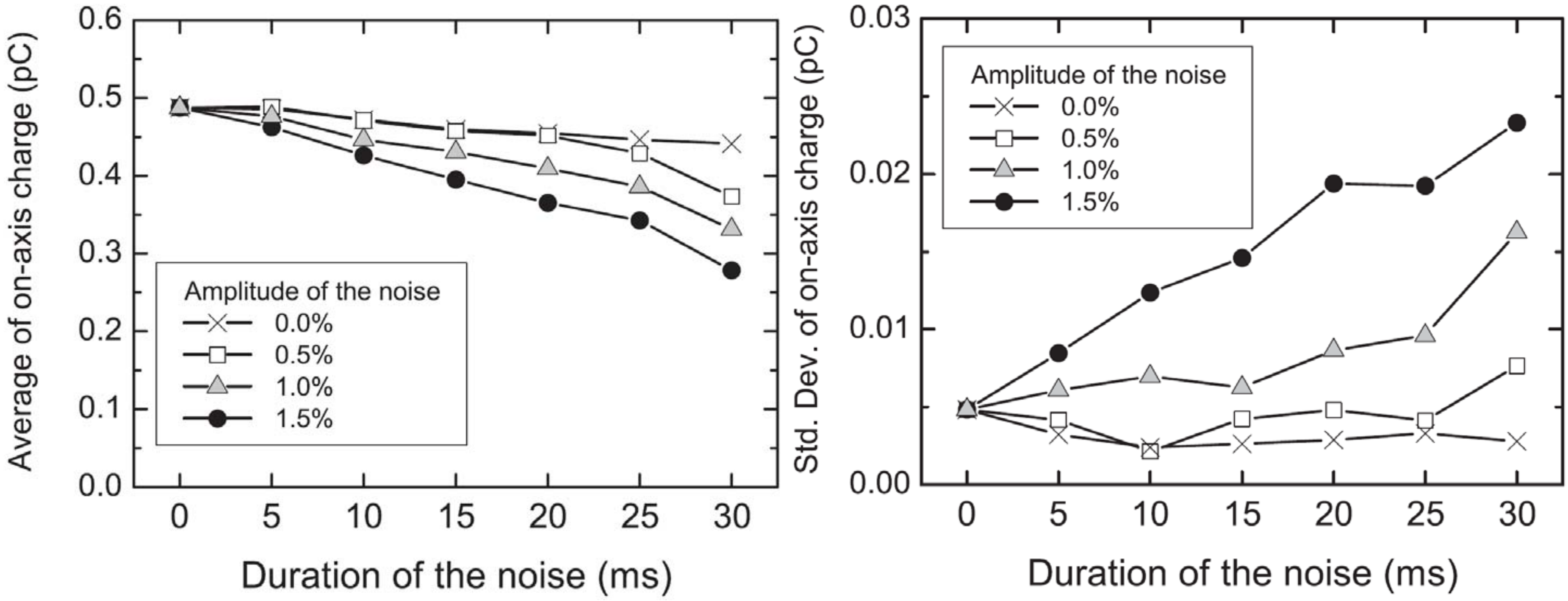
$$\frac{d^2 R_b}{dt^2} + \omega_q^2 R_b - \frac{Nq^2}{4\pi\varepsilon_0 m R_b} - \frac{\varepsilon}{4R_b^3} = 0$$





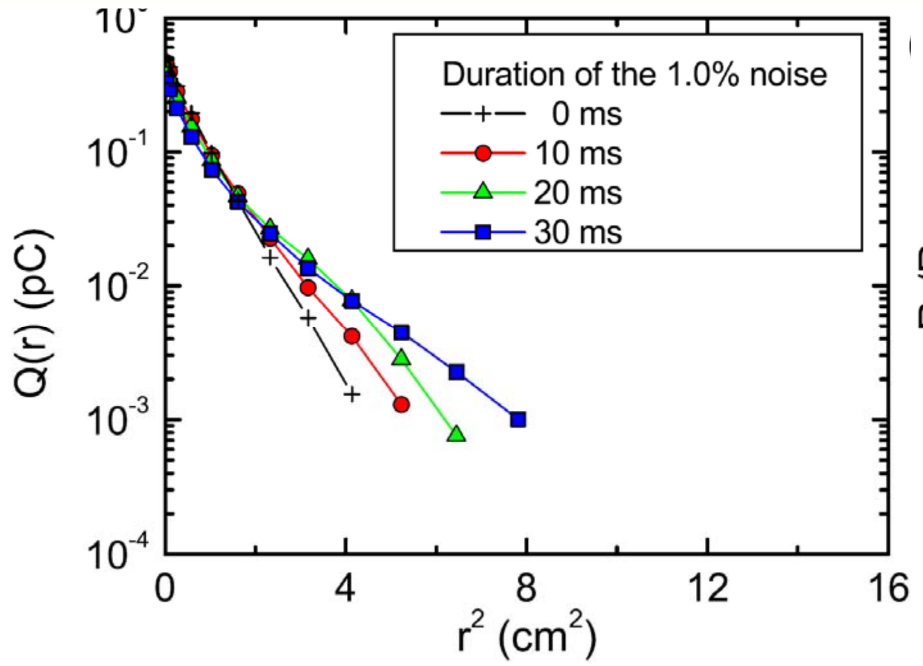
The Average On-axis Charge Decays Linearly and the Standard Deviation Grows with Noise Duration

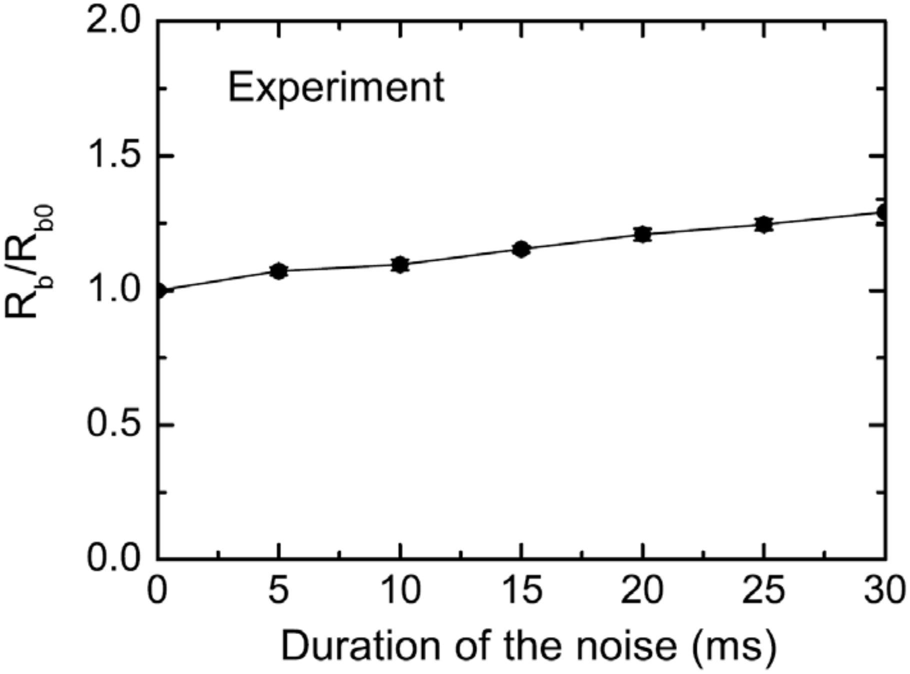
Average and standard deviation of 20 different error samples.



The standard deviation is ~ 5% of the average value

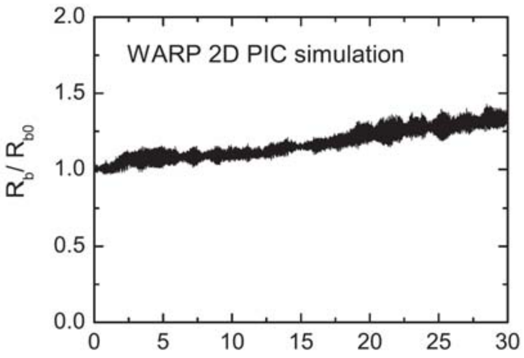
Noise Causes The Transverse Density Profile to Broaden and the RMS Radius to Grow Linearly with Time



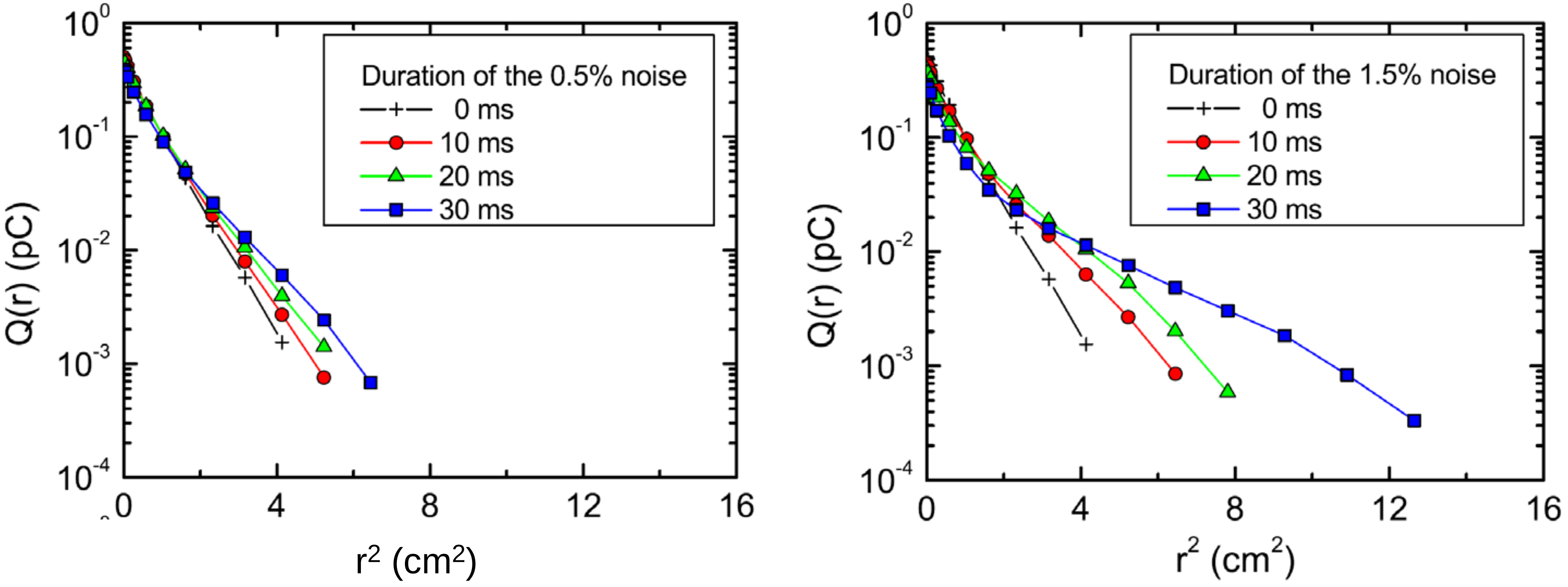


One error sample used

Compare to…



Broadening Depends on Noise Amplitude

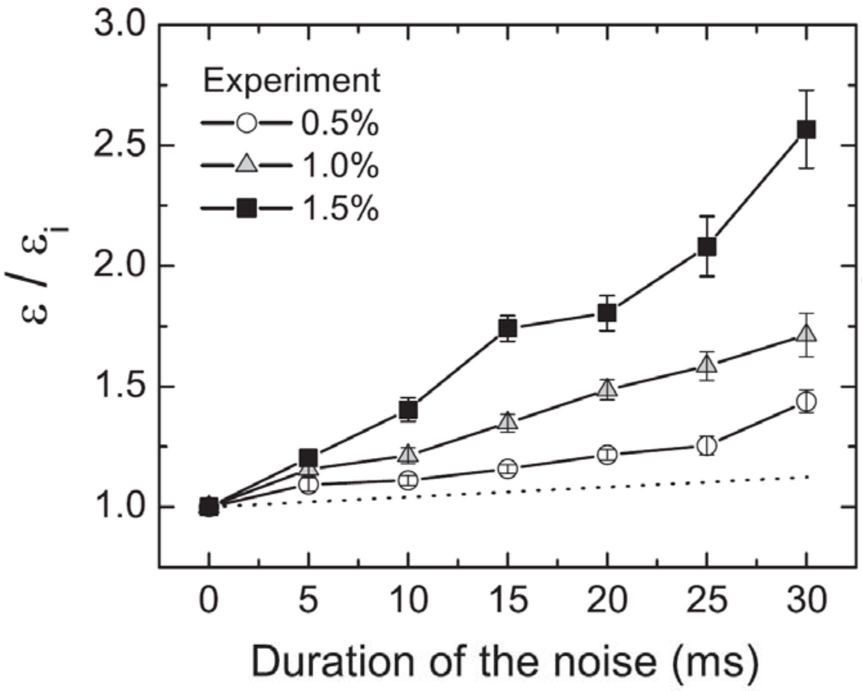


One error sample used

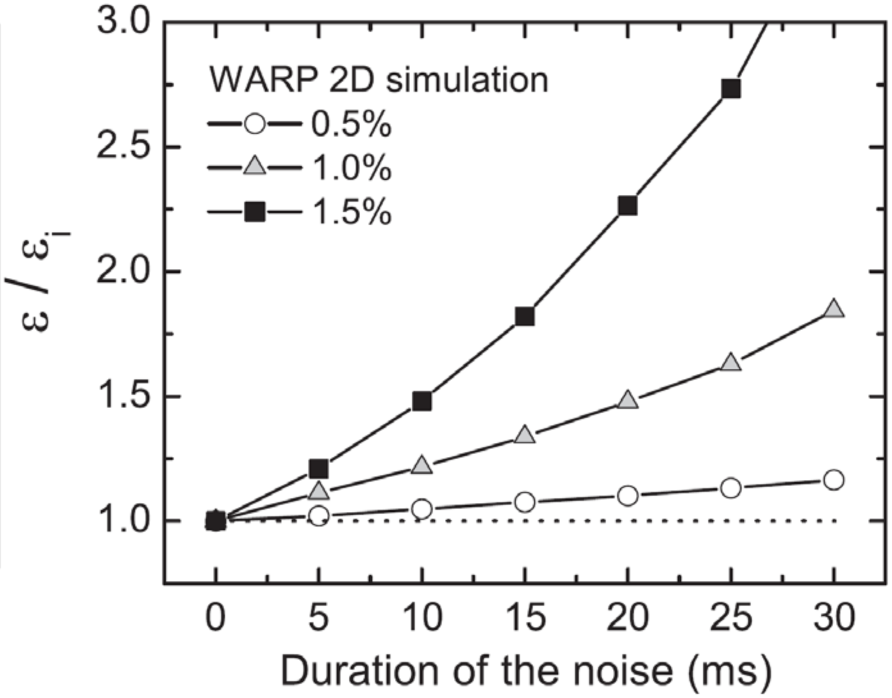
Even Small Amplitude Noise Causes Significant Emittance Growth

$$\epsilon(t) = 2R_b\left(\omega_q^2 R_b^2 - \frac{Nq^2}{4\pi\epsilon_0 m}\right)^{1/2}$$

If $R_b = 1 + \gamma t$, then $\varepsilon \sim (1 + \gamma t)^2$



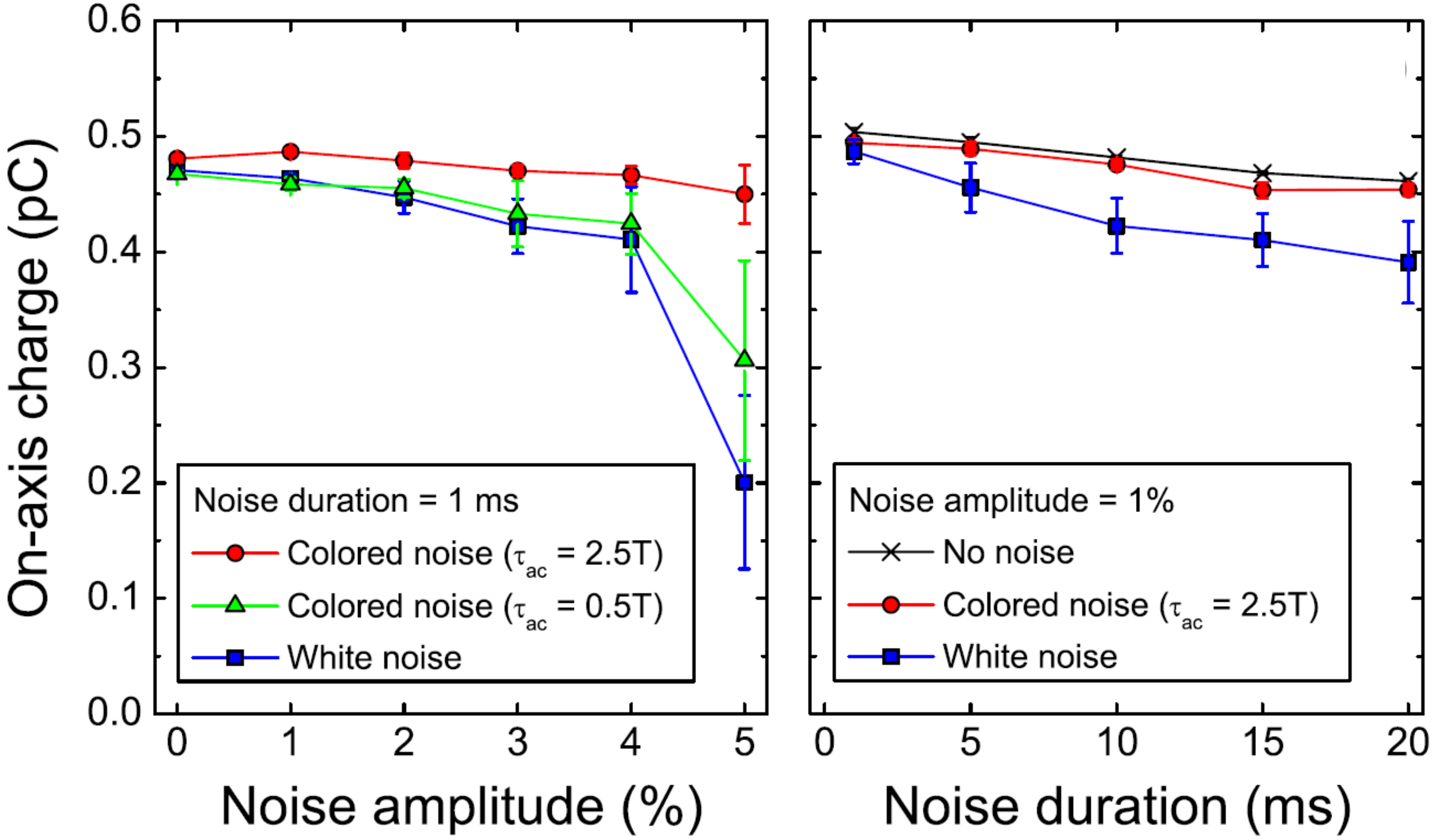
One error sample used



20 error samples used

White Noise Causes More Emittance Growth than Gaussian Colored Noise

$$\delta_{i+1} = \delta_i e^{-T/(2\tau_{ac})} + w_i \Delta_{\max}(1 - e^{-T/\tau_{ac}})^{1/2}$$

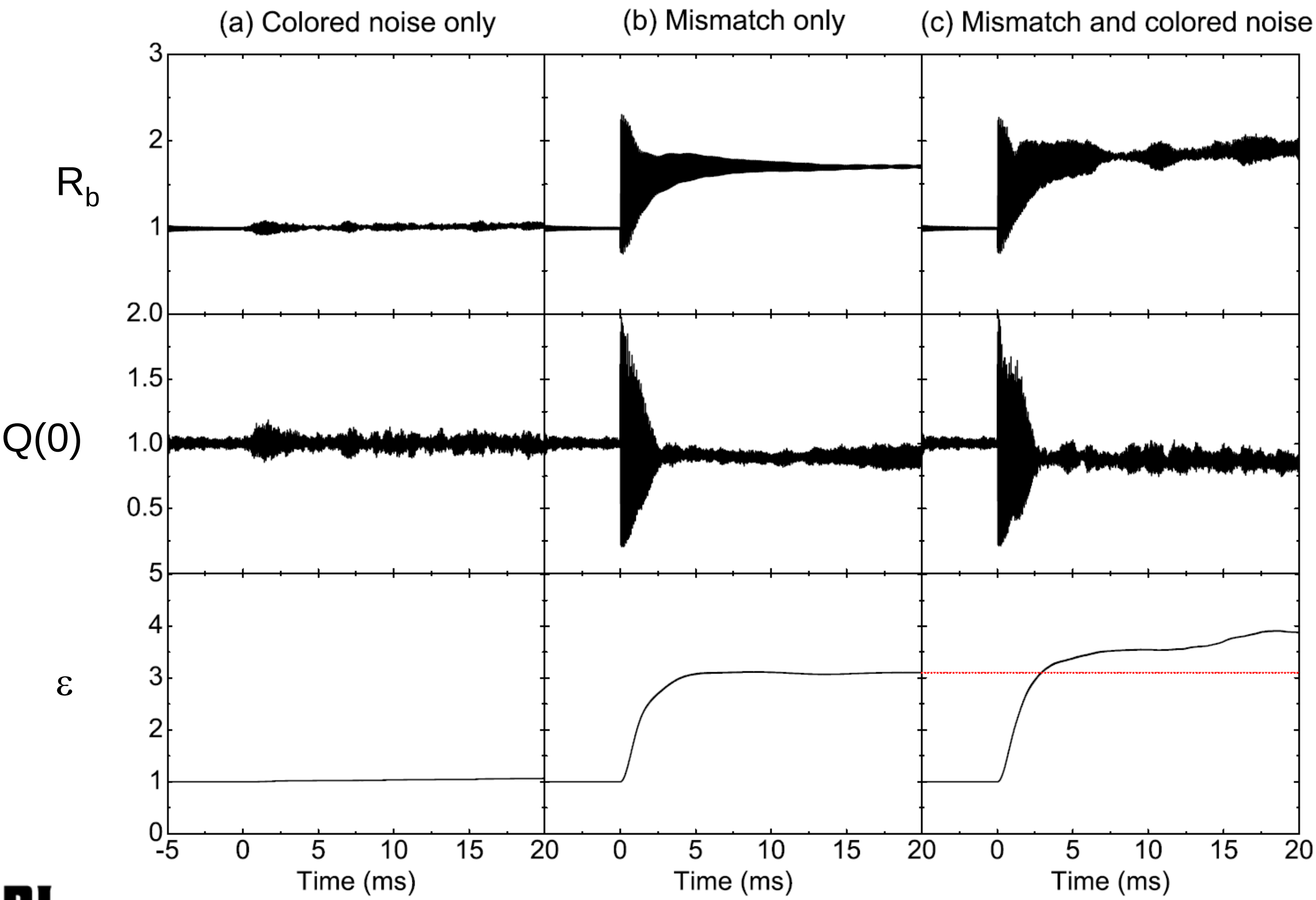


20 error samples used

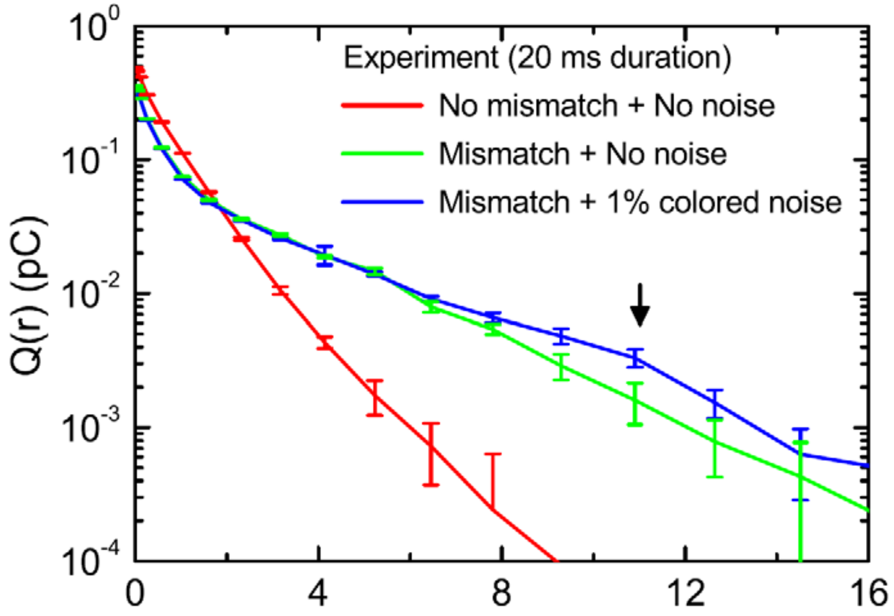
M. Chung *et al.*, Phys. Rev. ST Accel. Beams **12**, 054203 (2009).
I. Sideris and C. Bohn, Phys. Rev. ST Accel. Beams **7**, 104202 (2004).

Colored Noise with Large Autocorrelation Time Has Little Effect on Emittance Growth Without Mismatch Oscillations

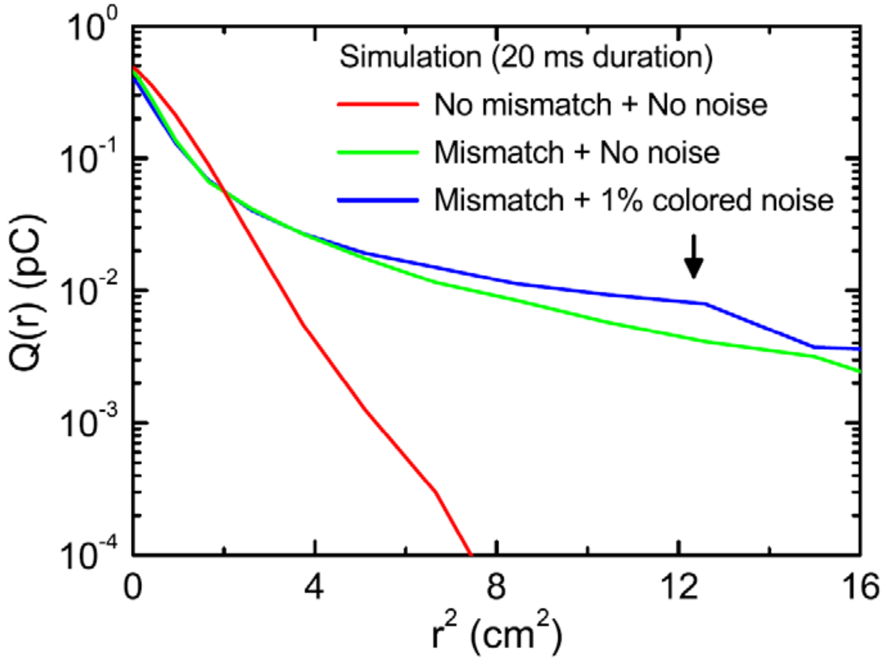
WARP 2D PIC Simulations, $\tau_{ac} = 5f^{-1}$



Experiments and Simulations Confirm Emittance Growth and Halo Particle Production when Beam Mismatch Oscillations and Colored Noise are Present



Experiments, $\tau_{ac} = 5f^{-1}$



WARP 2D PIC Simulations, $\tau_{ac} = 5f^{-1}$

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Transverse Bunch Compression by Adiabatically Increasing ω_q

$$\boxed{m\omega_q^2 R_b^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}}$$

If line density N is constant and if changes in kT are small, then increasing ω_q decreases R, and the bunch is compressed.

$$\boxed{\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi}$$

Either

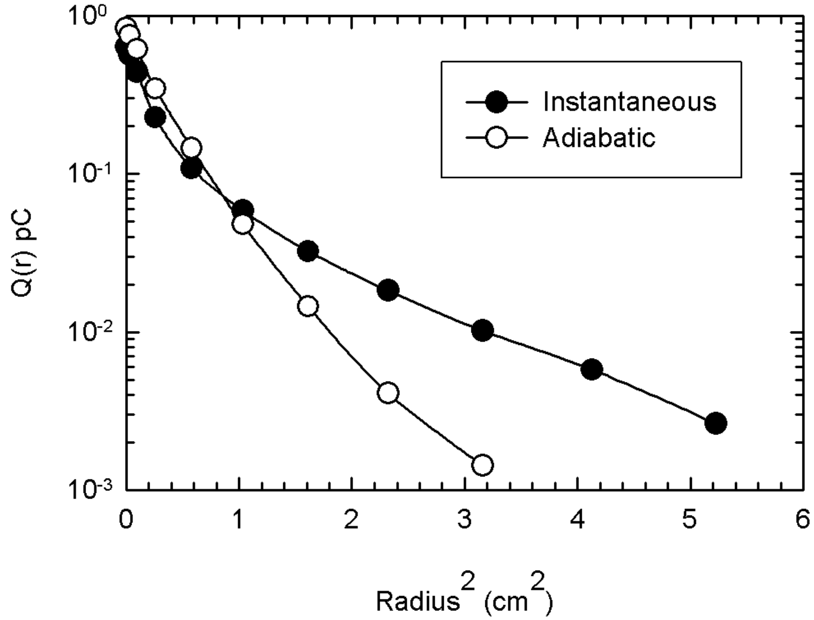
1.) increasing $V_{0\,\max}$ (increasing magnetic field strength) or

2.) decreasing f (increasing the magnet spacing)

increases ω_q

Adiabatic Amplitude Increases Transversely Compress the Bunch

90% increase in $V_{0\,\max}$



Baseline (not shown)	Instantaneous	Adiabatic
R_b = 0.83 cm	R_b = 0.93 cm	R_b = 0.63 cm
kT = 0.12 eV	kT = 0.58 eV	kT = 0.26 eV
s = 0.20	s = 0.08	s = 0.10
	$\Delta\varepsilon$ = 140%	$\Delta\varepsilon$ = 10%

Nucl. Inst. and Meth. in Phys. Res. **A 577**, 117 (2007).

Peak Density Scales Linearly With ω_q

$$m\omega_q^2 R^2 \sim 2kT$$

$$\varepsilon \sim R\sqrt{kT}$$

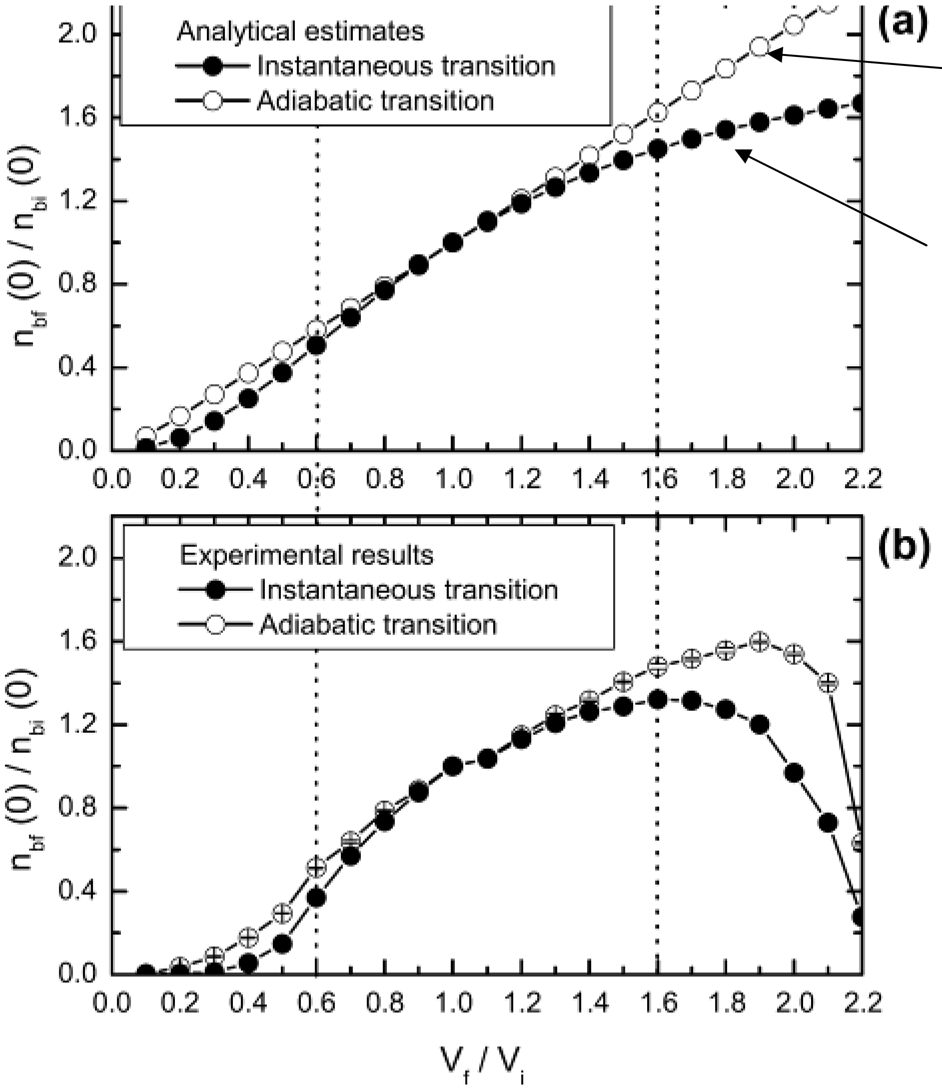
$$\Downarrow$$

$$\omega_q R^2 \sim const.$$

$$n(0)R^2 \sim N = const.$$

$$n(0) \sim \omega_q$$

$$\omega_q = \frac{8eV_{0\max}}{m\pi r_w^2 f}\xi$$



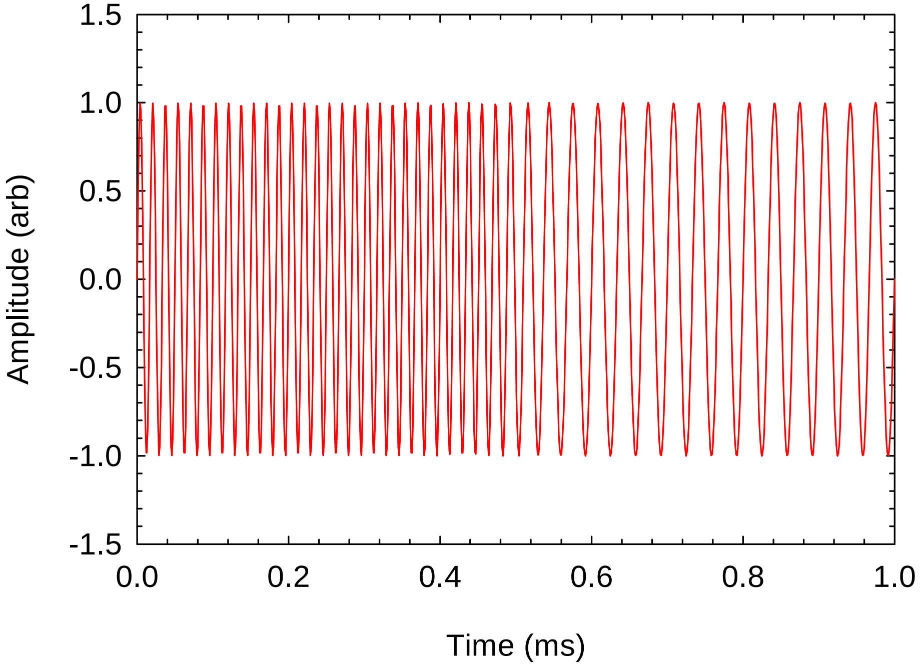
Constant emittance

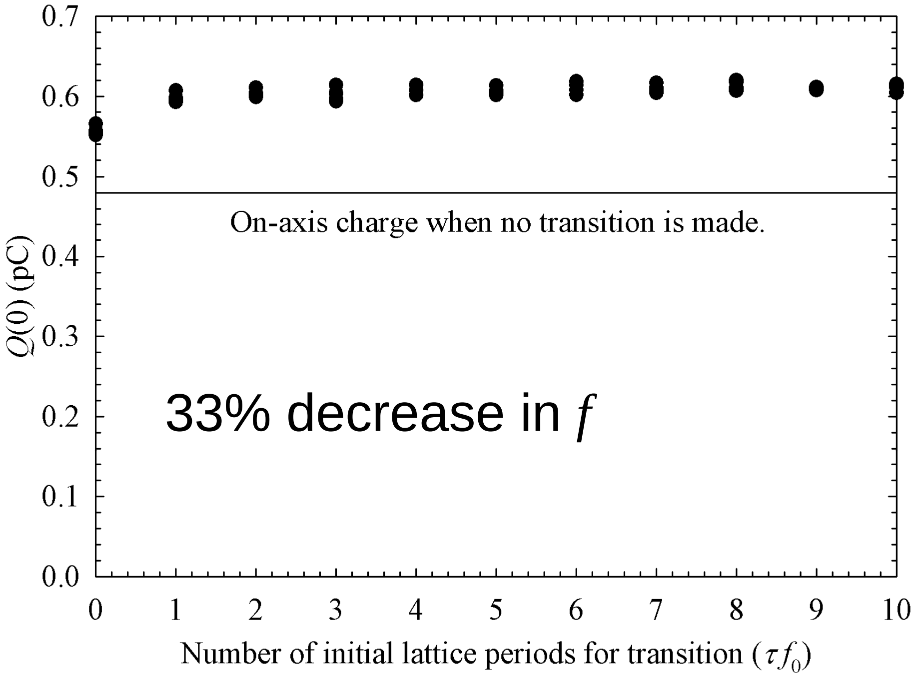
Constant energy

Phys. Rev. ST Accel. Beams **10**, 064202 (2007).

Increasing ω_q by Adiabatically Decreasing *f*

$$V(t) = V_{0\,\max} \sin \phi(t)$$





On-axis charge when no transition is made.

33% decrease in *f*

Phys. Rev. ST Accel. Beams **10**, 124201 (2007).

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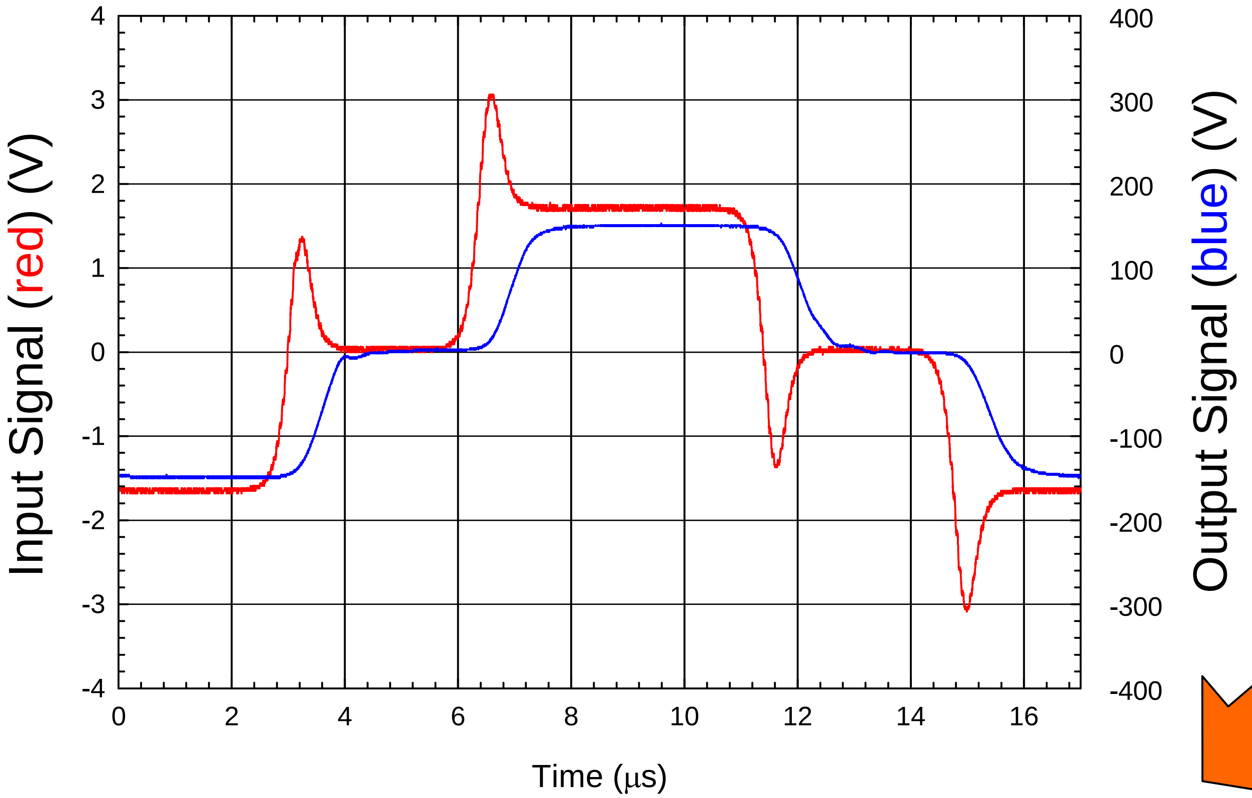
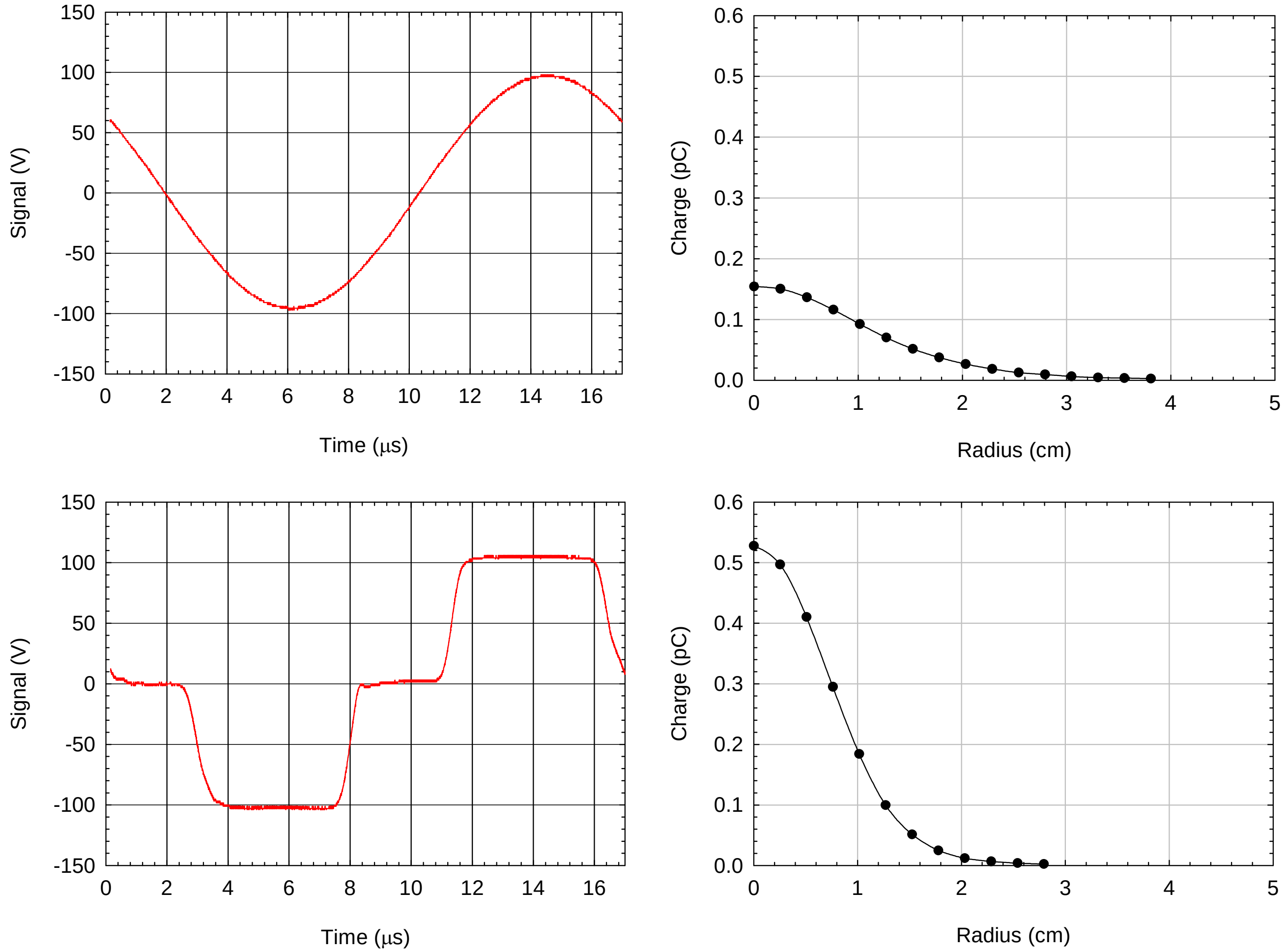
FODO Waveforms are More Realistic When Comparing PTSX Results to Transport Systems

The RC characteristic of the load require overdriving the driver circuits to apply FODO waveforms.



Tue. PM, M. Gutierrez, JP8.00061 : Application of Piecewise Continuous Waveforms in the Paul Trap Simulator Experiment

For Equal ω_q the FODO Waveform Gives a Plasma With Half the Emittance



<u>Sine</u>

R_b = 1.5 cm

kT = 0.17 eV

s = 0.16

<u>FODO</u>

R_b = 1.0 cm

kT = 0.08 eV

s = 0.45

$\varepsilon_{\mathrm{FODO}}/\varepsilon_{\mathrm{Sin}}$ = 0.45

Summary

- The analogy between the Paul Trap Simulator Experiment (PTSX) and alternating-gradient transport systems allows many topics in beam physics to be studied in a compact laboratory experiment.
- Small-amplitude random noise leads emittance growth and halo particle production over large propagation distances and colored noise in the presence of mismatch oscillations leads to emittance growth.
- Adiabatic changes in the average transverse focusing frequency ω_q can compress the beam with minimal emittance growth.
- Focusing-Off-Defocusing-Off (FODO) waveforms are a more realistic simulation of alternating-gradient transport systems and create plasmas in PTSX that have a higher normalized intensity and a lower emittance.